Advanced Microeconomics: Lecture Note 6

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1 Complete information:

The first-best level of effort will be achieved. The agent gets ex post full insurance.

2 Incomplete information with risk-neutral agent:

The first-best level of effort is still implemented. The agent gets ex ante full insurance.

3 Incomplete information with limited liability:

Only the limited liability constraint for the bad state may be binding. When the limited liability constraint for the bad state is binding, the principal has to pay limited liability rent when high production is realized to induce high effort.

4 Incomplete information with risk-averse agent:

The agent's expected utility is always zero, although he gets a risk premium. There are efficiency loses due to risk premium.

1 Sharecropping

5 Assume that the principal is a landlord and the agent is the landlord's tenant.

6 By exerting an effort e in $\{0,1\}$, the tenant increases (resp. decreases) the probability $\lambda(e)$ (resp. $1-\lambda(e)$) that a large q_H (resp. small q_L) quantity of an agricultural product is produced.

	q_H	q_L
$e=1$	λ_1	$1-\lambda_1$
$e=0$	λ_0	$1-\lambda_0$

7 The price of this good is normalized to one so that the principal's stochastic return on the activity is also q_H or q_L, depending on the state of nature.

8 It is often the case that peasants in developing countries are subject to strong financial constraints.

To model such a setting we assume that the agent is risk neutral and protected by limited liability.

9 When he wants to induce effort, the principal's optimal contract must solve

$$\begin{aligned}
&\underset{(t_H,t_L)}{\text{maximize}} && \lambda_1(q_H - t_H) + (1-\lambda_1)(q_L - t_L)\\
&\text{subject to} && \lambda_1 t_H + (1-\lambda_1)t_L - \psi \geq \lambda_0 t_H + (1-\lambda_0)t_L,\\
& && \lambda_1 t_H + (1-\lambda_1)t_L - \psi \geq 0,\\
& && t_L \geq 0.
\end{aligned}$$

10 LL and IC should be binding at the optimum. The optimal contract therefore satisfies $t_L^{\text{SB}} = 0$ and $t_H^{\text{SB}} = \frac{\psi}{\Delta\lambda}$.

11 The expected utilities obtained respectively by the principal and the agent are given by

$$\begin{aligned}
EV^{\text{SB}} &= \lambda_1(q_H - t_H^{\text{SB}}) + (1-\lambda_1)(q_L - t_L^{\text{SB}}) = \lambda_1 q_H + (1-\lambda_1)q_L - \lambda_1\frac{\psi}{\Delta\lambda},\\
EU^{\text{SB}} &= \lambda_1 t_H^{\text{SB}} + (1-\lambda_1)t_L^{\text{SB}} - \psi = \frac{\lambda_0\psi}{\Delta\lambda}.
\end{aligned}$$

Note that we need to assume that $EV^{\text{SB}} \geq \lambda_0 q_H + (1-\lambda_0)q_L$.

12 Contracts, however, often take the form of simple linear schedules linking the tenant's production to his compensation.

As an exercise, let us now analyze a simple linear sharing rule between the landlord and his tenant, with the landlord offering the agent a fixed share α of the realized production.

13 Such a sharing rule automatically satisfies the agent's limited liability constraint, which can therefore be omitted in what follows.

14 Formally, the optimal linear rule inducing effort must solve:

$$\begin{aligned}
&\underset{\alpha}{\text{maximize}} && (1-\alpha)\left[\lambda_1 q_H + (1-\lambda_1)q_L\right]\\
&\text{subject to} && \alpha\left[\lambda_1 q_H + (1-\lambda_1)q_L\right] - \psi \geq \alpha\left[\lambda_0 q_H + (1-\lambda_0)q_L\right],\\
& && \alpha\left[\lambda_1 q_H + (1-\lambda_1)q_L\right] - \psi \geq 0.
\end{aligned}$$

15 Obviously, only IC is binding at the optimum. One finds the optimal linear sharing rule to be

$$\alpha^{\text{SB}} = \frac{\psi}{\Delta\lambda\Delta q}.$$

16 Note that $\alpha^{\text{SB}} < 1$ because, for the agricultural activity to be a valuable venture in the first-best world, we must have $\Delta\lambda\Delta q > \psi$.

17 Hence, the return on the agricultural activity is shared between the principal and the agent, with highpowered incentives (α close to one) being provided

- when the disutility of effort is large or
- when the principal's gain from an increase in effort $\Delta\lambda\Delta q$ is small.

18 This sharing rule also yields the following expected utilities to the principal and the agent, respectively

$$EV_\alpha = (1-\alpha^{\text{SB}})\left[\lambda_1 q_H + (1-\lambda_1)q_L\right] = \left(1 - \frac{\psi}{\Delta\lambda\Delta q}\right)\left[\lambda_1 q_H + (1-\lambda_1)q_L\right],$$
$$EU_\alpha = \alpha^{\text{SB}}\left[\lambda_1 q_H + (1-\lambda_1)q_L\right] - \psi = \frac{\psi}{\Delta\lambda\Delta q}\left[\lambda_1 q_H + (1-\lambda_1)q_L\right] - \psi.$$

19 Comparing EV^{SB} and EV_α on the one hand and EU^{SB} and EU_α on the other hand, we observe that the constant sharing rule benefits the agent but not the principal.

$$EV_\alpha - EV^{\text{SB}} = -\frac{\psi}{\Delta\lambda\Delta q}q_L < 0,$$
$$EU_\alpha - EU^{\text{SB}} = \frac{\psi}{\Delta\lambda\Delta q}q_L > 0.$$

20 A linear contract is less powerful than the optimal second-best contract.

The former contract is an inefficient way to extract rent from the agent even if it still provides sufficient incentives to exert effort.

21 A linear sharing rule allows the agent to keep some strictly positive rent EU_α.

22 If the space of available contracts is extended to allow for fixed fees F, the principal can nevertheless bring the agent down to the level of his outside opportunity by setting a fixed

$$F^{\text{SB}} = \frac{\lambda_1 q^H + (1-\lambda_1)q_L}{\Delta q}\frac{\psi}{\Delta\lambda} - \psi.$$

2 Financial contracts

23 Assume that a risk-averse entrepreneur wants to start a project that requires an initial investment worth an amount I.

The entrepreneur has no cash of his own and must raise money from a bank or any other financial intermediary.

24 The return on the project is random and equal to V_H (resp. V_L), with probability $\lambda(e)$ (resp. $1-\lambda(e)$), where the effort exerted by the entrepreneur e belongs to $\{0,1\}$.

We denote the spread of profits by $\Delta V = V_H - V_L > 0$.

25 The financial contract consists of repayments (z_H, z_L), depending upon whether the project is successful or not.

26 To induce effort from the borrower, the risk-neutral lender's program is written as

$$\begin{aligned}
\underset{(z_H,z_L)}{\text{maximize}} \quad & \lambda_1 z_H + (1-\lambda_1)z_L - I \\
\text{subject to} \quad & \lambda_1 u(V_H - z_H) + (1-\lambda_1)u(V_L - z_L) - \psi \geq \lambda_0 u(V_H - z_H) + (1-\lambda_0)u(V_L - z_L), \\
& \lambda_1 u(V_H - z_H) + (1-\lambda_1)u(V_L - z_L) - \psi \geq 0.
\end{aligned}$$

where constraints are respectively the agent's incentive and participation constraints.

27 With the change of variables, $t_H = V_H - z_H$ and $t_L = V_L - z_L$, the principal's program takes its usual form.

$$\begin{aligned}
\underset{(t_H, t_L)}{\text{maximize}} \quad & \lambda_1(V_H - t_H) + (1-\lambda_1)(V_L - t_L) - I \\
\text{subject to} \quad & \lambda_1 u(t_H) + (1-\lambda_1)u(t_L) - \psi \geq \lambda_0 u(t_H) + (1-\lambda_0)u(t_L), \\
& \lambda_1 u(t_H) + (1-\lambda_1)u(t_L) - \psi \geq 0.
\end{aligned}$$

This change of variables also highlights the fact that everything happens as if the lender was benefitting directly from the return of the project, and then paying the agent only a fraction of the returns in the different states of nature.

28 Let us define the second-best cost of implementing a positive effort C^{SB}, and let us assume that $\Delta\lambda\Delta V \geq C^{\text{SB}}$, so that the lender wants to induce a positive effort level even in a second-best environment.

29 The lender's expected profit is worth

$$V_1 = \lambda_1 V_H + (1-\lambda_1)V_L - C^{\text{SB}} - I.$$

30 Only the projects with positive value $V_1 > 0$ will be financed. This requires the investment to be low enough, and typically we must have

$$I < I^{\text{SB}} = \lambda_1 V_H + (1-\lambda_1)V_L - C^{\text{SB}}.$$

31 Under complete information and no moral hazard, the project would instead be financed as soon as

$$I < I^* = \lambda_1 V_H + (1-\lambda_1)V_L - C^*.$$

32 For intermediary values of the investment, i.e., for I in $[I^{\text{SB}}, I^*]$, moral hazard implies that some projects are financed under complete information but no longer under moral hazard. This is akin to some form of credit rationing.

33 IC and IR should be binding at the optimum. Thus, the optimal contract is

$$t_H^{\text{SB}} = h\left(\psi + \tfrac{1-\lambda_1}{\Delta\lambda}\psi\right) > 0 \text{ and } t_L^{\text{SB}} = h\left(\psi - \tfrac{\lambda_1}{\Delta\lambda}\psi\right) < 0.$$

34 This optimal financial contract offered to the risk-averse and cashless entrepreneur does not satisfy the limited liability constraint $t \geq 0$.

To be induced to make an effort, the agent must bear some risk, which implies a negative payoff in the bad state of nature.

35 Adding the limited liability constraint (i.e., $t_L \geq 0$), the optimal contract would instead entail

$$t_H^{\text{LL}} = h(\tfrac{\psi}{\Delta\lambda}) \text{ and } t_L^{\text{LL}} = 0.$$

36 This contract has sometimes been interpreted in the corporate finance literature as a debt contract, which no money being left to the borrower in the bad state of nature and the residual being pocketed by the lender in the good state of nature.

37 Since h is strictly convex and $h(0) = 0$, we have that

$$t_H^{\text{LL}} - t_L^{\text{LL}} = h\left(\tfrac{\psi}{\Delta\lambda}\right) < h\left(\psi + \tfrac{1-\lambda_1}{\Delta\lambda}\psi\right) - h\left(\psi - \tfrac{\lambda_1}{\Delta\lambda}\psi\right) = t_H^{\text{SB}} - t_L^{\text{SB}}.$$

This inequality shows that the debt contract has less incentive power than the optimal incentive contract. It becomes harder to spread the agent's payments between both states of nature to induce effort if the agent is protected by limited liability.

3 Insurance contracts

38 Consider a risk-averse agent with utility function $u(\cdot)$ and initial wealth w.

39 With probability $\lambda(e)$ (resp. $1-\lambda(e)$) the agent has no (resp. an) accident and pays an amount z_H (resp. z_L) to an insurance company.

40 The damage incurred by the agent is worth d.

Effort e in $\{0,1\}$ can now be interpreted as a level of safety care.

41 The insurance company is assumed to be a monopoly and to have all of the bargaining power when offering the insurance contract to the insuree.

42 To induce effort from the insuree, the optimal insurance contract must solve:

$$\begin{aligned} \underset{(z_H,z_L)}{\text{maximize}} \quad & \lambda_1 z_H + (1-\lambda_1) z_L \\ \text{subject to} \quad & \lambda_1 u(w-z_H) + (1-\lambda_1)u(w-d-z_L) - \psi \geq \lambda_0 u(w-z_H) + (1-\lambda_0)u(w-d-z_L), \\ & \lambda_1 u(w-z_H) + (1-\lambda_1)u(w-d-z_L) - \psi \geq u(\hat{w}), \end{aligned}$$

where $u(\hat{w})$ is the quo utility when the agent does not purchase any insurance, which may not be zero.

43 $\hat{w}$ is the certainty equivalent of the agent's wealth when he does not purchase any insurance and when he exerts an effort, which is implicitly defined as

$$u(\hat{w}) = \lambda_1 u(w) + (1-\lambda_1)u(w-d) - \psi.$$

Here we assume that the agent wants to exert an effort in the absence of an insurance contract, i.e.,

$$\lambda_1 u(w) + (1-\lambda_1)u(w-d) - \psi > \lambda_0 u(w) + (1-\lambda_0)u(w-d),$$

or

$$u(w) - u(w-d) > \tfrac{\psi}{\Delta\lambda}.$$

One could assume instead that he does not want to exert effort when he is not insured. In this case, his quo utility level would be $\lambda_0 u(w) + (1-\lambda_0)u(w-d) = u(\hat{w}')$.

44 Letting

$$t_H = w - z_H \text{ and } t_L = w - d - z_L$$

and

$$S_H = w \text{ and } S_L = w - d,$$

Except for the nonzero reservation value, the problem becomes familiar:

$$\begin{aligned}
\underset{(t_H,t_L)}{\text{maximize}} \quad & \lambda_1(w-t_H)+(1-\lambda_1)(w-d-t_L) \\
\text{subject to} \quad & \lambda_1 u(t_H)+(1-\lambda_1)u(t_L)-\psi \geq \lambda_0 u(t_H)+(1-\lambda_0)u(t_L), \\
& \lambda_1 u(t_H)+(1-\lambda_1)u(t_L)-\psi \geq u(\hat{w}).
\end{aligned}$$

45 Both constraints are again binding at the optimum, and the optimal contract is

$$\begin{aligned}
t_H^{\text{SB}} &= h\left(\psi+u(\hat{w})+(1-\lambda_1)\frac{\psi}{\Delta\lambda}\right), \\
t_L^{\text{SB}} &= h\left(\psi+u(\hat{w})-\lambda_1\frac{\psi}{\Delta\lambda}\right).
\end{aligned}$$

46 The second-best cost of inducing effort is now written as

$$\begin{aligned}
C^{\text{SB}}(\hat{w}) &= \lambda_1 t_H^{\text{SB}}+(1-\lambda_1)t_L^{\text{SB}} \\
&= \lambda_1 h\left(\psi+u(\hat{w})+(1-\lambda_1)\frac{\psi}{\Delta\lambda}\right)+(1-\lambda_1)h\left(\psi+u(\hat{w})-\lambda_1\frac{\psi}{\Delta\lambda}\right).
\end{aligned}$$

47 Without moral hazard this cost of inducing effort would instead be

$$C^*(\hat{w}) = h\left(\psi+u(\hat{w})\right),$$

where $t_H^* = t_L^* = t^* = \psi+u(\hat{w})$.

48 Let $AC(\hat{w})$ denote the agency cost, which is the difference between the second-best and the first-best cost of inducing effort.

That is, let $AC(\hat{w}) = C^{\text{SB}}(\hat{w})-C^*(\hat{w})$ be the agency cost due to moral hazard incurred by the principal.

49 Differentiating with respect to $\hat{w}$, we have

$$\begin{aligned}
AC'(\hat{w}) &= \lambda_1 h'\left(\psi+u(\hat{w})+(1-\lambda_1)\frac{\psi}{\Delta\lambda}\right)u'(\hat{w})+(1-\lambda_1)h'\left(\psi+u(\hat{w})-\lambda_1\frac{\psi}{\Delta\lambda}\right)u'(\hat{w}) \\
&\quad - h'\left(\psi+u(\hat{w})\right)u'(\hat{w}) \\
&= u'(\hat{w})\left[\lambda_1 h'\left(\psi+u(\hat{w})+(1-\lambda_1)\frac{\psi}{\Delta\lambda}\right)+(1-\lambda_1)h'\left(\psi+u(\hat{w})-\lambda_1\frac{\psi}{\Delta\lambda}\right)-h'\left(\psi+u(\hat{w})\right)\right].
\end{aligned}$$

Since h' is strictly convex, $AC'(\hat{w})>0$. That is, $AC(\hat{w})$ is monotonically increasing with $\hat{w}$.

Interpretation: When the agent's wealth increases, the agency cost increases since there is more distortion due to moral hazard in the decision of the insurance company to induce effort or not.

Task

- Reading: 4.8 in [LM], 5.4 in [S].
- Understanding: