Advanced Microeconomics: Lecture Note 15

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1 Herding

1. Herding is the act of changing individual behavior to follow a trend.

   - Bubble
   - New fashion trend

   Question: Why do people follow suit and cluster on the same (occupational, cultural, consumption) choice? 为什么人们会跟随潮流并在同一种（职业、文化、消费）选择上形成聚集？

2. There are many possible answers: similarity in taste, social conformity, positive network externality. 有许多可能的答案：口味相似、社会从众、积极的网络外部性。

3. The theory of informational cascades is a theory that explains fads, fashion, custom and cultural changes from the perspective of observational learning. 信息级联效应理论是一种从观察学习的角度解释时尚、流行、习俗和文化变迁的理论。

   Bikhchandani, Hirshleifer, and Welch (JPE, 1992) look at the particular angle of observational learning and they offer a rational justification of people's incentive to abandon private opinion and follow the crowd, even if the crowd's choice is not necessarily correct. Simply, the theory of informational cascades rationalizes herding behavior as an outcome of observational learning. 该理论将从众行为解释为观察学习的结果。

   信息级联效应是指在决策过程中，个体往往会依赖他人的行为或决策来做出自己的决策，而忽视自身拥有的信息和判断能力。当一个人观察到其他人的行为或选择后，他可能会放弃自己的判断，而盲目地追随他人的选择，即使这些选择可能与他个人的信息和偏好不一致。

   这种效应在信息不完全或不确定的情况下尤为显著，导致信息的集中和传统观点的传播，而忽略了个体的独立思考和判断能力。信息级联效应可以导致信息的扭曲和市场失灵，影响决策的效率和结果。

2 Setup

4. There are two new restaurants in town, A and B.

   Which restaurant is better is uncertain with equal prior probability, i.e., true state is either A or B with

   $$\text{Prob}(A) = \text{Prob}(B) = \tfrac{1}{2}.$$

5. There are an infinite sequence of consumers who arrive one by one.

   Each consumer i receives an i.i.d. private signal $s_i \in \{s_A, s_B\}$.

- s_A means "I personally think restaurant A is better";
- s_B means "I personally think restaurant B is better".

6. The accuracy of the private signal is $p > \frac{1}{2}$:

$$\text{Prob}(s_A \mid A) = \text{Prob}(s_B \mid B) = p > \tfrac{1}{2} \text{ and } \text{Prob}(s_A \mid B) = \text{Prob}(s_B \mid A) = 1 - p < \tfrac{1}{2}.$$

7. Consumers have identical payoff functions: Each consumer gets payoff 1 if he chooses the better restaurant and 0 otherwise.

   Tie-breaking rule: When a consumer is indifferent, he flips a coin. That is, he chooses restaurant A with probability $\frac{1}{2}$.

8. Timeline of the game:

   - The first consumer arrives and picks a restaurant based on his private signal s_1. Denote his choice as C_1.
   - The second consumer observes the choice of the first consumer as well as his private signal s_2, and picks a restaurant C_2.
   - For any $k \geq 2$, the k-th consumer observes the choices of all previous consumers, $C_1, C_2, \ldots, C_{k-1}$, as well as his private signal s_k. Based on these information, he picks a restaurant C_k.

3 Decision rule

9. Each consumer has two types: The private signal can be either s_A or s_B.

10. Each consumer's strategy is a function from the "set of histories $\times$ the set of types" to $\{A, B\}$.

    Given the history of the game, consumer i's strategy is a pair (C_i^A, C_i^B), where C_i^A denotes his choice if his signal is s_A and C_i^B denotes his choice if the signal is s_B.

11. Perfect Bayesian equilibrium requires each consumer i always chooses the best option according to his belief about the true state.

    Suppose that, conditional on all available information, consumer i believes that restaurant A is better with probability x.

    Then the expected utilities are

$$\begin{aligned} \mathbf{E}[u(A)] &= u(\text{choose A when A is better}) \cdot x + u(\text{choose A when B is better}) \cdot (1 - x) = x, \\ \mathbf{E}[u(B)] &= u(\text{choose B when A is better}) \cdot x + u(\text{choose B when B is better}) \cdot (1 - x) = 1 - x. \end{aligned}$$

    Thus, $\mathbf{E}[u(A)] > \mathbf{E}[u(B)]$ when $x > \frac{1}{2}$, and $\mathbf{E}[u(A)] < \mathbf{E}[u(B)]$ when $x < \frac{1}{2}$.

12. Proposition: The decision rule for each consumer is:

$$\begin{cases} \text{choose } A, & \text{if Prob}(A \mid \text{current information}) > \frac{1}{2}, \\ \text{choose } B, & \text{if Prob}(A \mid \text{current information}) < \frac{1}{2}, \\ \text{choose } A \text{ with prob } 0.5, & \text{if Prob}(A \mid \text{current information}) = \frac{1}{2}. \end{cases}$$

4 Belief updating

13. Perfect Bayesian equilibrium also require that each consumer correctly update his belief about the true state using Bayes' rule.

4.1 The first consumer

14. The first consumer observes only his private signal s_1, so he updates his belief according to the realization of his signal:

$$\begin{aligned}\mathrm{Prob}(A \mid s_A) &= \frac{\mathrm{Prob}(s_A \mid A)\,\mathrm{Prob}(A)}{\mathrm{Prob}(s_A)} = \frac{\frac{1}{2}p}{\mathrm{Prob}(s_A)},\\ \mathrm{Prob}(A \mid s_B) &= \frac{\mathrm{Prob}(s_B \mid A)\,\mathrm{Prob}(A)}{\mathrm{Prob}(s_B)} = \frac{\frac{1}{2}(1-p)}{\mathrm{Prob}(s_B)}.\end{aligned}$$

Since

$$\begin{aligned}\mathrm{Prob}(s_A) &= \mathrm{Prob}(s_A \mid A) \times \mathrm{Prob}(A) + \mathrm{Prob}(s_A \mid A) \times \mathrm{Prob}(A) = \tfrac{1}{2}p + \tfrac{1}{2}(1-p) = \tfrac{1}{2},\\ \mathrm{Prob}(s_B) &= \mathrm{Prob}(s_B \mid A) \times \mathrm{Prob}(A) + \mathrm{Prob}(s_B \mid A) \times \mathrm{Prob}(A) = \tfrac{1}{2}(1-p) + \tfrac{1}{2}p = \tfrac{1}{2},\end{aligned}$$

we have

$$\mathrm{Prob}(A \mid s_A) = p > \tfrac{1}{2} \text{ and } \mathrm{Prob}(A \mid s_B) = 1 - p < \tfrac{1}{2}.$$

15. Based on the decision rule, the first consumer always chooses according to his private signal, i.e., his strategy is

$$(C_1^A, C_1^B) = (A, B).$$

4.2 The second consumer

16. When the second consumer arrives, he observes not only his private signal s_2 but also the choice of the first consumer, C_1.

    Moreover, the second consumer also knows that the choices of the first consumer perfectly reveals his private signal according to his strategy $(C_1^A, C_1^B) = (A, B)$.

    Based on these information, he updates his belief in each of the four possible scenarios.

17. Case 1: When $C_1 = A$ and $s_2 = s_A$,

$$\begin{aligned}\mathrm{Prob}(A \mid C_1 = A, s_2 = s_A) &= \mathrm{Prob}(A \mid s_1 = s_A, s_2 = s_A)\\ &= \frac{\mathrm{Prob}(s_1 = s_A, s_2 = s_A \mid A)\,\mathrm{Prob}(A)}{\mathrm{Prob}(s_1 = s_A, s_2 = s_A)}\\ &= \frac{\mathrm{Prob}(s_1 = s_A, s_2 = s_A \mid A)\,\mathrm{Prob}(A)}{\mathrm{Prob}(s_1 = s_A, s_2 = s_A \mid A)\,\mathrm{Prob}(A) + \mathrm{Prob}(s_1 = s_A, s_2 = s_A \mid B)\,\mathrm{Prob}(B)}\\ &= \frac{p^2\frac{1}{2}}{p^2\frac{1}{2} + (1-p)^2\frac{1}{2}}\\ &= \frac{p^2}{p^2 + (1-p)^2} > \frac{p^2}{p^2 + p^2} = \frac{1}{2}.\end{aligned}$$

Therefore, the second consumer strictly prefers to choose A.

18. Similarly, when $C_1 = A$ and $s_2 = s_B$, we can use the Bayes' rule to get

$$\text{Prob}(A \mid C_1 = A, s_2 = s_B) = \frac{1}{2}.$$

This means that the second consumer is indifferent between A and B and will flip a coin.

19. When $C_1 = B$ and $s_2 = s_A$,

$$\text{Prob}(A \mid C_1 = B, s_2 = s_A) = \frac{1}{2}.$$

The second consumer is indifferent between A and B and will flip a coin.

20. Finally, when $C_1 = B$ and $s_2 = s_B$,

$$\text{Prob}(A \mid C_1 = B, s_2 = s_B) = \frac{(1-p)^2}{(1-p)^2 + p^2} < \frac{1}{2},$$

and the second consumer strictly prefers to choose B.

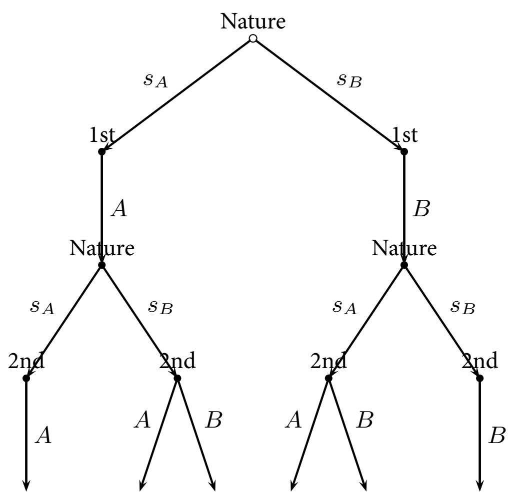
21. Based on these four scenarios, the strategy of the second consumer is conditional on the action of the first consumer:

- If the first consumer chose A, then

$$(C_2^A, C_2^B) = \big(A, \tfrac{1}{2} \circ A + \tfrac{1}{2} \circ B\big).$$

- If the first consumer chose B, then

$$(C_2^A, C_2^B) = \big(\tfrac{1}{2} \circ A + \tfrac{1}{2} \circ B, B\big).$$



4.3 The third consumer

22. When the third consumer arrives, he observes both choices from consumer 1 and 2 as well as his private signal s_3.

There are four possible realizations of choices from the first two consumers:

$$(C_1, C_2) \in \{(A, A), (A, B), (B, A), (B, B)\}.$$

To simplify things, let's classify them into two categories.

23. Case 1: $C_1 \neq C_2$.

    (a) If the choice of the second consumer differs from the choice of the first, then it must be the case that the second consumer's private signal is opposite of the first one's.

    (b) In this case, the choices of the first consumers perfectly reveals their (opposite) private signals. That is,

    $$(C_1, C_2) = (A, B) \Longrightarrow (s_1, s_2) = (s_A, s_B),$$
    $$(C_1, C_2) = (B, A) \Longrightarrow (s_1, s_2) = (s_B, s_A).$$

    (c) Since the consumers' signals are equally accurate, the effect of a pair of opposite signals cancel out:

    $$\text{Prob}(A \mid s_A, s_B) = \text{Prob}(A \mid s_B, s_A) = \tfrac{1}{2}.$$

    (d) Therefore, when $C_1 \neq C_2$, the third consumer cannot learn anything from the choices of the first two consumers and he has to rely his choice solely on his own private signal.

    (e) In other words, when the first pair of consumers differ in their choices, we can simply delete them from the sequence. The third consumer acts as if he is the first consumer in the sequence; he chooses according to his private signal only: $(C_3^A, C_3^B) = (A, B)$.

24. Case 2: $C_1 = C_2$.

25. Suppose $C_1 = C_2 = A$ and $s_3 = s_A$.

$$\begin{aligned}
& \text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_A) \\
= \; & \text{Prob}(A \mid s_1 = s_3 = s_A, s_2 = s_A \text{ or } s_2 = s_B \text{ but the coin flip suggests } A) \\
= \; & \frac{\text{Prob}(s_1 = s_3 = s_A, s_2 = s_A \text{ or } s_2 = s_B \text{ but the coin flip suggests } A \mid A)\,\text{Prob}(A)}{\text{Prob}(s_1 = s_3 = s_A, s_2 = s_A \text{ or } s_2 = s_B \text{ but the coin flip suggests } A)} \\
= \; & \frac{p^2[p + \frac{1}{2}(1-p)]\frac{1}{2}}{p^2[p + \frac{1}{2}(1-p)]\frac{1}{2} + (1-p)^2[(1-p) + \frac{1}{2}p]\frac{1}{2}} \\
= \; & \frac{p^2[p + \frac{1}{2}(1-p)]}{p^2[p + \frac{1}{2}(1-p)] + (1-p)^2[(1-p) + \frac{1}{2}p]}.
\end{aligned}$$

Since $p > \frac{1}{2}$, we have $p^2 > (1-p)^2$ and $p + \frac{1}{2}(1-p) = \frac{1}{2} + \frac{1}{2}p > 1 - \frac{1}{2}p = (1-p) + \frac{1}{2}p$.

Then $p^2[p + \frac{1}{2}(1-p)]\frac{1}{2} > (1-p)^2[(1-p) + \frac{1}{2}p]$, and hence $\text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_A) > \frac{1}{2}$.

Therefore, the third consumer strictly prefers to choose A.

Similar result holds for the case when $C_1 = C_2 = B$ and $s_3 = s_B$.

In summary, the third consumer continues to receive a private signal that suggests the same action, he will again choose the same action $C_3 = C_1 = C_2$.

26. Suppose $C_1 = C_2 = A$ and $s_3 = s_B$.

The third consumer updates his belief:

$$\text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_B) = \frac{\text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid A)\,\text{Prob}(A)}{\text{Prob}(C_1 = C_2 = A, s_3 = s_B)},$$

where

$$
\begin{aligned}
\text{Prob}(C_1 = C_2 = A, s_3 = s_B) = \; & \text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid A)\,\text{Prob}(A) \\
& + \text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid B)\,\text{Prob}(B), \\
\text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid A) = \; & \text{Prob}(s_1 = s_2 = s_A, s_3 = s_B \mid A) + \tfrac{1}{2}\text{Prob}(s_1 = s_A, s_2 = s_3 = s_B \mid A) \\
= \; & p^2(1-p) + \tfrac{1}{2}p(1-p)^2, \\
\text{Prob}(C_1 = C_2 = A, s_3 = s_B \mid B) = \; & \text{Prob}(s_1 = s_2 = s_A, s_3 = s_B \mid B) + \tfrac{1}{2}\text{Prob}(s_1 = s_A, s_2 = s_3 = s_B \mid B) \\
= \; & (1-p)^2 p + \tfrac{1}{2}(1-p)p^2.
\end{aligned}
$$

Therefore,

$$
\text{Prob}(A \mid C_1 = C_2 = A, s_3 = s_B) = \frac{p^2(1-p) + \frac{1}{2}p(1-p)^2}{p^2(1-p) + \frac{1}{2}p(1-p)^2 + (1-p)^2 p + \frac{1}{2}(1-p)p^2} = \frac{1+p}{3} > \frac{1}{2},
$$

the last inequality is due to $p > \frac{1}{2}$.

Based on the decision rule, this means that, if the first two consumers both chose restaurant A, the third consumer will choose restaurant A even if his private signal is s_B.

27. The third consumer will follow suit and choose A regardless of his private signal. This is called an informational cascade of A.

5 Informational cascade

28. A player is said to be in an informational cascade of action X if he chooses X regardless of his private signal. 信息级联表现为许多人以连续的方式做出相同的决定。信息级联的产生通常需要两个步骤，第一个步骤是开始级联。这是一个决策过程，通常为二元决策。第二步是后续的人根据自己的外部信息和其他人的行为做出自己的决策。

29. Question: If the third consumer is choosing A regardless of his private signal, what about the other consumers?

30. Answer: All of them choose A regardless of their private signals.

    Intuition:

    - If the third consumer's choice is irrelevant of his private signal, his choice becomes completely uninformative to the fourth consumer because C_3 reveals nothing about s_3.
    - Therefore, the fourth consumer faces the exact same decision problem as the third consumer; he chooses A regardless of his private signal, too.
    - The choice of the fourth consumer is uninformative to the fifth.
    - The fifth faces the same problem as the third; again, he chooses A regardless of his signal.
    - …
    - The same result applies for every consumer later in the (infinite) sequence.

31. Theorem: If player k is in an informational cascade of action X, every player $i > k$ is in the information cascade of action X.

32. Summary: In the perfect Bayesian equilibrium of this game

- A pair of choices $(A, A) \Longrightarrow$ an informational cascade of A;
- A pair of choices $(B, B) \Longrightarrow$ an informational cascade of B;
- A pair of choices (A, B) or $(B, A) \Longrightarrow$ delete this uninformative pair and restart the game.

33. When a cascade occurs, the choice of infinitely many consumers is, in fact, determined by the private signals of only two consumers.

34. When there are infinite consumers, eventually we will encounter a pair of (A, A) or (B, B) in the sequence. Thus, in the limit, a cascade occurs with probability 1.

35. Because a cascade is essentially triggered by only one pair of identical signals, a small change in the information environment can easily change the formation of a cascade.

36. For example: If there is an expert (whose signal is more accurate than the others) in the sequence:
    - If this expert shows up after a cascade already occurred, he may be able to overturn the current trend.
    - If this expert is the first player in the sequence, then a cascade starts immediately.

37. Because cascades stop social learning, delaying cascades can expose more private signals and improve social welfare.

    For example, if there is a tiny percentage of anonymous laypeople (whose signals are less accurate than normal people) in the population:

    (a) Overall signal accuracy of the population decreases slightly.

    (b) But normal people do not jump into a cascade so quickly $\Longrightarrow$ enhanced social learning.

    (c) In general, a tiny percentage of laypeople can help decrease Prob$(incorrect)$; see Wu (JEBO 2015).

Task

- Reading:
- Understanding: