Advanced Microeconomics: Lecture Note 8

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1 When two parties engage in a relationship, it is often the case that they are uncertain about the value of some parameter that will affect their future gains from trade. This uncertainty is represented by assuming that the parameter can take several values, each value corresponding to different states of nature whose probability distribution is common knowledge.

Even though they will both learn the value of the parameter in the future, the trading partners cannot write ex ante contracts contingent on the state of nature, because this state of nature is not verifiable by a third party that could enforce their contract. That is the nonverifiability (不可验证性) of the state of nature.

不可验证性问题在现实中并不少见：代理人有私有信息，但由于某些原因（比如两者都对某所在的行业比较了解），委托人也可以观察到该信息。但双方都不可能对此提供客观证据，而外界又不掌握了解这一行业所需的专业知识，从而无法验证私有信息。

2 The goal is to assess whether the nonverifiability significantly affects the ability of the contractual partners to realize the full gains from trade.

Result preview: When the principal has the full ability to commit to a mechanism at the ex ante stage and a benevolent court of law is available, the nonverifiability of the state of nature alone does not create transaction costs.

3 A principal wishes to hire an agent to run a one-time project.

If the agent's effort level is $q \in [0, \infty)$, then principal's income is $S(q)$, with $S(0) = 0$, $S'(q) > 0$, and $S''(q) < 0$ for all q.

If the principal pays transfer t to the agent, then his utility is $S(q) - t$.

4 The agent is an expected utility maximizer with utility $t - C(q, \theta)$.

- $\theta \in \{\theta_L, \theta_H\}$ represents agent's marginal cost. Here, $\theta_H > \theta_L$ and $\text{Prob}(\theta_L) = \lambda \in (0, 1)$.
- $C(q, \theta)$ measures the disutility/cost of effort.
- $C(0, \theta) = 0$, $C_q > 0$, $C_\theta > 0$, $C_{qq} > 0$, $C_{q\theta} > 0$, $C_{qq\theta} > 0$.

$\Rightarrow$ The agent's indifference curves have single-crossing property.

- The agent is risk neutral and has a reservation utility 0.

5 The first-best outcomes are assumed to be interior:

- $S'(q_L^*) = C_q(q_L^*, \theta_L)$ when θ_L.
- $S'(q_H^*) = C_q(q_H^*, \theta_H)$ when θ_H.

We also assume that $S(q_L^*) - C(q_L^*, \theta_L) > 0$ and $S(q_H^*) - C(q_H^*, \theta_H) > 0$. That is, delegation is valuable.

1 No ex ante contract (ex post bargaining)

6 We consider the case where the principal and the agent do not write any contract ex ante. Bargaining over the gains from trade takes place ex post, i.e., once the state of nature is commonly known.

1.1 Principal has full bargaining power

7 We assume that the principal has all the bargaining power at the ex post stage.

8 The sequence of play is as follows:

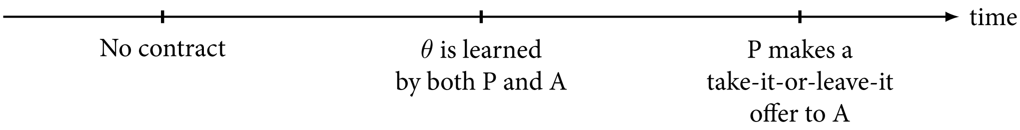


Figure 1: Timing

After being informed about θ, the principal can make a take-it-or-leave-it offer (reflecting his full bargaining power) to the agent under complete information.

9 The offer can implement the first-best outcome:

- If agent is of high ability θ_L, then he will produce q_L^* such that $S'(q_L^*) = C_q(q_L^*, \theta_L)$, receive payment $t_L^* = C(q_L^*, \theta_L)$.
- If agent is of low ability θ_H, then he will produce q_H^* such that $S'(q_H^*) = C_q(q_H^*, \theta_H)$, receive payment $t_H^* = C(q_H^*, \theta_H)$.

1.2 Bargaining with equal weights

10 We assume that the principal and the agent have equal weights in the negotiation at the ex post stage.

11 The sequence of play is as follows:

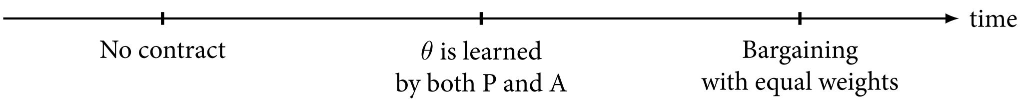


Figure 2: Timing

12 We use Nash bargaining solution (纳什议价解/纳什谈判解).

13 A two-person bargaining problem, denoted by $\langle U, d \rangle$, consists of

- U is the set of possible agreements in terms of utilities that they yield to 1 and 2. An element of U is a pair $u = (u_1, u_2)$.
- d is a pair (d_1, d_2), called the disagreement point or threat point.

If agreement $u = (u_1, u_2) \in U$ is reached, then 1 gets utility u_1 and 2 gets utility u_2. If no agreement is reached then 1 gets utility d_1 and 2 gets utility d_2.

The set of two-person bargaining games is denoted by W.

14 Convention: Assume that

- U is compact and convex.
- U contains a point y for which $y_i > d_i$ for $i = 1, 2$, that is, bargaining is worthwhile for both the players.

15 The Nash bargaining solution is a mapping $f\colon W \to \mathbb{R}^2$ that associates a unique element $f(U,d)$ with the game $\langle U, d\rangle$, satisfying the following axioms:

N1. Feasibility: $f(U,d) \in U$.

N2. Individual rationality: $f(U,d) \geq d$ for all $\langle U, d\rangle \in W$.

N3. Pareto optimality: $f(U,d)$ is Pareto optimal. That is, there does not exist a point $(u_1, u_2) \in U$ such that

$$u_1 \geq f_1(U,d),\ u_2 \geq f_2(U,d),\ (u_1,u_2) \neq f(U,d).$$

N4. Symmetry: If $\langle U, d\rangle \in W$ satisfies $d_1 = d_2$ and $(x_1, x_2) \in U$ implies $(x_2, x_1) \in U$, then $f_1(U,d) = f_2(U,d)$.

N5. Invariance under linear transformations: Let $a_1, a_2 > 0$, $b_1, b_2 \in \mathbb{R}$, and $\langle U, d\rangle, \langle U', d'\rangle \in W$ where $d_i' = a_i d_i + b_i$, $i = 1, 2$, and $U' = \{x \in \mathbb{R}^2 \mid x_i = a_i y_i + b_i, i = 1, 2, y \in U\}$. Then $f_i(U_i', d_i') = a_i f_i(U,d) + b_i$, $i = 1, 2$.

N6. Independence of irrelevant alternatives: If $\langle U, d\rangle, \langle U', d'\rangle \in W$, $d = d'$, $U \subseteq U'$, and $f(U', d') \in U$, then $f(U,d) = f(U', d')$.

The interpretation is that, given any bargaining problem $\langle U, d\rangle$, the solution function tells us that the agreement $u = f(U,d)$ will be reached.

16 Theorem: A game $\langle U, d\rangle \in W$ has a unique Nash solution $u^* = f(U,d)$ satisfying Conditions N1 to N6. Furthermore, the solution u^* satisfies Conditions N1 to N6 if and only if

$$(u_1^* - d_1)(u_2^* - d_2) > (u_1 - d_1)(u_2 - d_2)$$

for all $u \in U$, $u \geq d$, and $u \neq u^*$.

17 Remark:

- Existence of an optimal solution: Since the set U is compact and the objective function is continuous, there exists an optimal solution.
- Uniqueness of the optimal solution: The objective function is strictly quasi-concave. Therefore, maximization problem has a unique optimal solution.

18 When the agent is of high ability, they shall agree on output q and payment t, which are Nash solutions to the problem

$$\max_{(q,t)} \big(S(q) - t\big)\big(t - C(q, \theta_L)\big).$$

19 Solution:

- Payment $t_L^{\text{NB}} = \frac{1}{2}\big[S(q_L^*) + C(q_L^*, \theta_L)\big]$.
- Output is the first-best one q_L^* such that $S'(q_L^*) = C_q(q_L^*, \theta_L)$;

Both principal and agent receive an equal share of the first-best gains.

20 Similarly for the low ability case.

- Payment $t_H^{\text{NB}} = \frac{1}{2}\left[S(q_H^*) + C(q_H^*, \theta_H)\right]$.
- Output is the first-best one q_H^* such that $S'(q_H^*) = C_q(q_H^*, \theta_H)$;

Both principal and agent receive an equal share of the first-best gains.

21 Summary:

- Bargaining over the gains from trade takes place ex post, i.e., once the state of nature is commonly known.
- If the principal has all the bargaining power ex post, the first-best outcome is implemented, with the agent being maintained at his status quo utility level.
- If we had considered a more even distribution of the bargaining power ex post, outcome efficiency would still be preserved, but the distribution of the gains from trade would be more egalitarian: the principal (resp. the agent) would obtain a lower (resp. higher) utility level.
- If the principal does not expect to have all the bargaining power at the ex post stage, he strictly prefers to design a mechanism at the ex ante stage when he still has all the bargaining power. (next section)

2 Ex ante contract

22 Instead of waiting for the realization of the state of nature, the principal can offer to the agent, at the ex ante stage, a menu of contracts.

23 The contract can only be written in terms of the verifiable variables. θ is not verifiable and cannot be written into a contract.

A menu $\{(q_L, t_L), (q_H, t_H)\}$ (equivalently, a nonlinear payment $t(q)$) is a feasible instrument.

- When facing the menu $\{(q_L, t_L), (q_H, t_H)\}$, agent accepts the menu itself or not.
- In contrast, in the standard model of adverse selection, agent chooses (q_L, t_L), (q_H, t_H), or neither when he faces a menu $\{(q_L, t_L), (q_H, t_H)\}$.

When agent accepts such a contract $\{(q_L, t_L), (q_H, t_H)\}$, the agent anticipates that

- his choice of outputs q_L in state θ_L will satisfy the following interim constraint

$$t_L - C(q_L, \theta_L) \geq t_H - C(q_H, \theta_L).$$

- his choice of outputs q_H in state θ_H will satisfy the following interim constraint

$$t_H - C(q_H, \theta_H) \geq t_L - C(q_L, \theta_H).$$

These constraints are the same as the standard incentive compatibility constraints as in adverse selection.

24 The sequence of play is as follows:

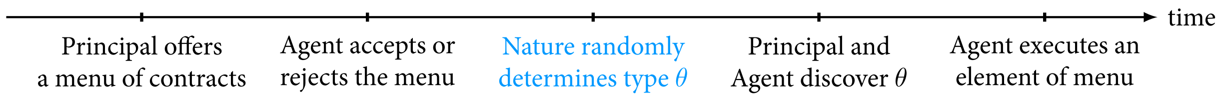


Figure 3: Timing

25 Agent's problem:

- (ex ante) IR: $\lambda\left(t_L - C(q_L,\theta_L)\right) + (1-\lambda)\left(t_H - C(q_H,\theta_H)\right) \geq 0$.
- IC: $t_L - C(q_L,\theta_L) \geq t_H - C(q_H,\theta_L)$ and $t_H - C(q_H,\theta_H) \geq t_L - C(q_L,\theta_H)$.

26 Principal's problem:

$$\begin{aligned} \underset{(q_L,t_L),(q_H,t_H)}{\text{maximize}} \quad & \lambda\left[S(q_L) - t_L\right] + (1-\lambda)\left[S(q_H) - t_H\right] \\ \text{subject to} \quad & \text{IR and IC.} \end{aligned}$$

27 IR should be binding at the optimum. Otherwise, principal can lower t_H and t_L simultaneously.

28 Ignoring IC, principal's problem is

$$\max_{q_L,q_H} \lambda\left[S(q_L) - C(q_L,\theta_L)\right] + (1-\lambda)\left[S(q_H) - C(q_H,\theta_H)\right].$$

SOC and FOC imply that $S'(q_L^*) = C_q(q_L^*,\theta_L)$ and $S'(q_H^*) = C_q(q_H^*,\theta_H)$.

29 IC conditions can be satisfied by setting

$$t_L^* = S(q_L^*) - T^* \text{ and } t_H^* = S(q_H^*) - T^*,$$

where T^* is a lump-sum payment:

$$\begin{aligned} t_L^* - t_H^* = S(q_L^*) - S(q_H^*) = \int_{q_H^*}^{q_L^*} S'(q)\,\mathrm{d}q \geq \int_{q_H^*}^{q_L^*} C_q(q,\theta_L)\,\mathrm{d}q = C(q_L^*,\theta_L) - C(q_H^*,\theta_L), \\ t_L^* - t_H^* = S(q_L^*) - S(q_H^*) = \int_{q_H^*}^{q_L^*} S'(q)\,\mathrm{d}q \leq \int_{q_H^*}^{q_L^*} C_q(q,\theta_H)\,\mathrm{d}q = C(q_L^*,\theta_H) - C(q_H^*,\theta_H). \end{aligned}$$

To make IR bind, we can choose $T^* = \lambda\left[S(q_L^*) - C(q_L^*,\theta_L)\right] + (1-\lambda)\left[S(q_H^*) - C(q_H^*,\theta_H)\right]$.

30 This implementation of the first-best outcome amounts to having the principal selling the benefit of the relationship to the risk-neutral agent for a fixed up-front payment T^*. The agent benefits from the full value of the good and trades off the value of any production against its cost just as if he was an efficiency maximizer.

31 Question: What are the similarities and differences between this ex ante contract and the selling contract in moral hazard?

32 Summary: Efficiency is always achieved when the single crossing property is satisfied for the agent's objective function:

- The first-best outcome can be implemented: $S'(q_L^*) = C_q(q_L^*,\theta_L)$ and $S'(q_H^*) = C_q(q_H^*,\theta_H)$,

$$t_L^* = S(q_L^*) - T^* \text{ and } t_H^* = S(q_H^*) - T^*,$$

where $T^* = \lambda\left[S(q_L^*) - C(q_L^*,\theta_L)\right] + (1-\lambda)\left[S(q_H^*) - C(q_H^*,\theta_H)\right]$.

33 Question: If agent is risk-averse, ex ante contracting fails to achieve efficiency.

3 Nash implementation

34 The principal and agent can achieve ex post efficiency through an ex ante contract when they are both risk neutral. This contract uses only agent's message but fails to achieve efficiency when the agent is risk-averse.

35 Consider the following mechanism:

- If both principal and agent report that θ_L has realized, the contract (q_L^*, t_L^*) is enforced, where

$$S'(q_L^*) = C_q(q_L^*, \theta_L) \text{ and } t_L^* = C(q_L^*, \theta_L).$$

- If both principal and agent report that θ_H has realized, the contract (q_H^*, t_H^*) is enforced, where

$$S'(q_H^*) = C_q(q_H^*, \theta_H) \text{ and } t_H^* = C(q_H^*, \theta_H).$$

- If they disagree, then nothing is enforced.

		Principal	
		θ_L	θ_H
Agent	θ_L	(q_L^*, t_L^*)	$(0,0)$
	θ_H	$(0,0)$	(q_H^*, t_H^*)

Figure 4

Note that the same game form must be played by the agent and the principal, whatever the true θ.

The goal of this mechanism is to ensure that there exists a truthful Nash equilibrium in each θ that implements the first-best outcome.

36 Proposition: The first-best outcome can be implementable in Nash equilibrium.

Proof. First consider θ_L.

- Given that agent reports θ_L, principal gets $S(q_L^*) - t_L^*$ by reporting the truth and zero otherwise.
- By assumption, the delegation is valuable: $S(q_L^*) - t_L^* = S(q_L^*) - C(q_L^*, \theta_L) > 0$.
- Telling the truth is a best response for principal.
- Agent is indifferent telling the truth or not when principal reports θ_L.

Next consider θ_H.

- Given that agent reports θ_H, principal gets $S(q_H^*) - t_H^*$ by reporting the truth and zero otherwise.
- By assumption, the delegation is valuable: $S(q_H^*) - t_H^* = S(q_H^*) - C(q_H^*, \theta_H) > 0$.
- Telling the truth is a best response for principal.
- Agent is indifferent telling the truth or not when principal reports θ_H.

□

37 New problem: When θ_L realizes, (θ_L, θ_L) is not the unique Nash equilibrium.

- Given that agent reports θ_H, principal gets $S(q_H^*) - t_H^* > 0$ by reporting θ_H and zero otherwise.
- Given that principal reports θ_H, agent gets $t_H^* - C(q_H^*, \theta_L)$ by reporting θ_H and zero otherwise. Notice that $t_H^* = C(q_H^*, \theta_H) > C(q_H^*, \theta_L)$.
- Thus, (θ_H, θ_H) is another Nash equilibrium.

Remark: When θ_H realizes, (θ_H, θ_H) is the unique Nash equilibrium.

38 Consider the following mechanism:

		Principal	
		θ_L	θ_H
Agent	θ_L	(q_L^*, t_L^*)	$(\hat{q}_2, \hat{t}_2)$
	θ_H	$(\hat{q}_1, \hat{t}_1)$	(q_H^*, t_H^*)

Figure 5

39 The outcomes $(\hat{q}_1, \hat{t}_1)$ and $(\hat{q}_2, \hat{t}_2)$ may be different from the no-trade option used above, in order to give more flexibility to the court in designing off-the equilibrium punishments, ensuring both the truthful revelation and the uniqueness of the equilibrium. Let us now see how it is possible to do so.

40 The conditions for having a truthful Nash equilibrium in θ_L are:

- For principal, reporting θ_L is better than reporting θ_H: $S(q_L^*) - t_L^* > S(\hat{q}_2) - \hat{t}_2$. (红线左上方)
- For agent, reporting θ_L is better than reporting θ_H: $0 = t_L^* - C(q_L^*, \theta_L) > \hat{t}_1 - C(\hat{q}_1, \theta_L)$. (2 号线右下方)

Similarly, the conditions for having a truthful Nash equilibrium in θ_H are:

- For principal, reporting θ_H is better than reporting θ_L: $S(q_H^*) - t_H^* > S(\hat{q}_1) - \hat{t}_1$. (蓝线左上方)
- For agent, reporting θ_H is better than reporting θ_L: $0 = t_H^* - C(q_H^*, \theta_H) > \hat{t}_2 - C(\hat{q}_2, \theta_H)$. (4 号线右下方)

41 Since $S(q_H^*) - t_H^* > S(\hat{q}_1) - \hat{t}_1$, principal still prefers to report θ_H when θ_L.

Thus, when θ_L, to ensure (θ_H, θ_H) not to be a Nash equilibrium, we must have: agent prefers to report θ_L:

$$\hat{t}_2 - C(\hat{q}_2, \theta_L) > t_H^* - C(q_H^*, \theta_L).$$

(5 号线左上方)

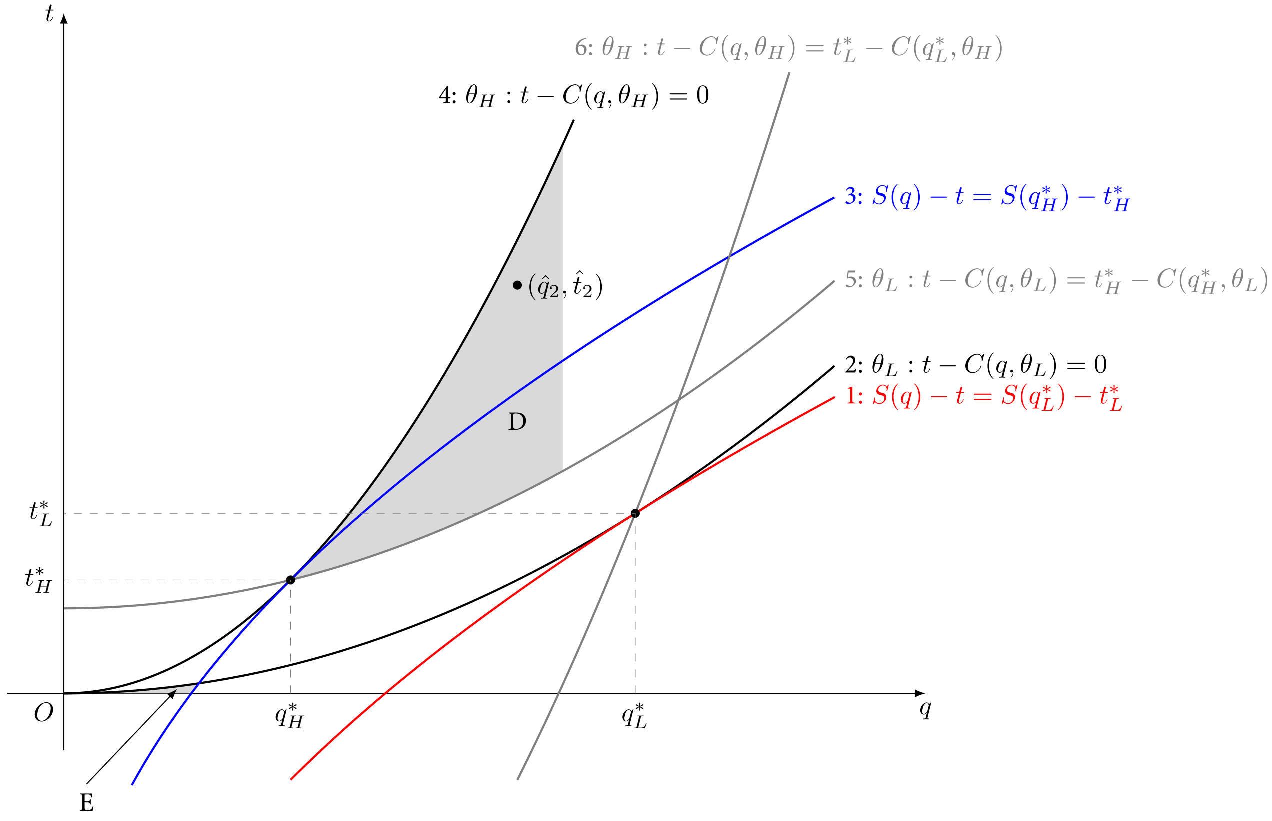
Similarly, when θ_H, to ensure (θ_L, θ_L) not to be a Nash equilibrium, we must have: agent prefers to report θ_H:

$$\hat{t}_1 - C(\hat{q}_1, \theta_H) > t_L^* - C(q_L^*, \theta_H).$$

(6 号线左上方)

42 Proposition: The first-best outcome can be uniquely implementable in Nash equilibrium.

Proof. Consider the following graph.



Pick $(\hat{q}_1, \hat{t}_1)$ in region E and $(\hat{q}_2, \hat{t}_2)$ in region D. □

43 Summary:

- The principal offers a mechanism that is designed to ensure that the noncooperative play of the game by both the principal and the agent yields the desired first-best allocation.
- In playing such a two-agent mechanism, the principal and the agent adopt a Nash behavior. An allocation rule is implementable in Nash equilibrium if there exists a mechanism and a Nash equilibrium of this mechanism where the agents follow strategies that induce the desired allocation in each state of the world.
- The standard principal-agent models are such that the first-best is implementable in Nash equilibrium with rather simple mechanisms.

Task

- Reading: 6.1–6.3 in [LM] (required), 6 in [陈] (required).
- Understanding: